Technical Comments

AIAA 82-4202

Comment on "Prediction of Range and Endurance of Jet Aircraft at Constant Altitude"

Seppo Laine*
Helsinki University of Technology, Espoo, Finland

IN his Engineering Note,[1] Professor Bert has made an error in calculating the value of the coefficient a_1 in Eq. (13). The value of a_1 should be 6.160×10^{-8} lb-s/ft^2 instead of 0.6160×10^{-8} lb-s/ft^2. Therefore his conclusions concerning an optimum velocity are incorrect. In fact, starting from Eq. (12) and using simple calculus we can obtain an expression for the optimum velocity ($V_{\rm opt}$) which maximizes the value of the specific range (R_s):

$$V_{\rm opt} = \sqrt{6a_{-3}} / (\sqrt{a_{-1}^2 + 12a_{-3}a_1} - a_{-1})^{1/2} \tag{1}$$

This formula can be converted into a more convenient form, which resembles the classical formula for $V_{\rm opt}$:

$$V_{\rm opt} = {}^4\!\sqrt{3}\, V_{\rm md} [\sqrt{1 + (C_0/C_2)^2} + C_0/C_2]^{1/2} \tag{2}$$

Here $V_{\rm md}$ is the airspeed at which the drag is a minimum, C_0 is the zeroth-order thrust coefficient of fuel consumption, and C_2 is an abbreviation for the expression $2\sqrt{3}\,\sqrt{KC_{D_0}}C_1 W$. The first part of the formula, i.e. ${}^4\!\sqrt{3}\, V_{\rm md}$, is simply the optimum velocity according to classical theory, in which the coefficient C_0 is assumed to be zero,[2] and the expression in the brackets accounts for the influence of the coefficient C_0. Utilizing the values of Professor Bert's example ($W = 2305$ lb, $\rho = 0.4623 \times 10^{-3}$ slug/ft^3, $C_{D_0} = 0.0154$, $K = 0.0471$, $C_0 = 22$ lb/h, $C_1 = 0.623$ 1/h) we obtain

$$V_{\rm opt} = 1.316 \times 417.6 \times 1.085 \text{ ft/s} = 596 \text{ ft/s}$$

The corresponding range R is 3559 mi, which also coincides with the overall maximum value for R better than ± 0.1 mi. Here the ratio $V_{\rm opt}/V_{\rm md}$ equals 1.428 and is 1.085 times higher than the value given by classical theory. This differs from the result arrived at by Bert, who, based on an erroneous numerical value for a_1, comes to the conclusion that the optimum velocity should equal the maximum velocity. In his numerical example $V_{\rm opt} = 1.864 V_{\rm md} = 779$ ft/s, which yields 3323 mi as the range, a value which is much lower than that given by the optimum velocity in this Comment.

Received Dec. 7, 1981; revision received March 12, 1982.

*Professor of Aeronautical Engineering. Member AIAA.

References

[1] Bert, C.W., "Prediction of Range and Endurance of Jet Aircraft at Constant Altitude," *Journal of Aircraft*, Vol. 18, Oct. 1981, pp. 890-892.

[2] Miele, A., *Flight Mechanics*, Addison-Wesley, Reading, Mass., 1962, p. 160.

AIAA 82-4203

Reply by Author to S. Laine

Charles W. Bert*
University of Oklahoma, Norman, Okla.

THE author wishes to express his gratitude to Professor Laine not only for his interest in the subject Engineering Note but also for finding the numerical error in the value of a_1 in Eq. (13) of the optimum design example and for extending the design usefulness of the work by deriving his new equation for the optimum velocity to maximize specific range.

First, the numerical value of a_1 was indeed incorrect, for two reasons: the omission of the value of the wing area (S, 100 ft^2) and a decimal error. The value of 6.160×10^{-8} lb-s/ft^2 mentioned by Professor Laine is correct. Of course, this error changed the character, as well as the numerical values, for the remainder of the author's design example.

The author agrees with Professor Laine's equations and his values of 596 ft/s for $V_{\rm opt}$ and 3559 mi. for the corresponding range.

It is instructive to note that the value of 3559 mi. corresponding to $V = V_{\rm opt} = 596$ ft/s is only 1.66% larger than the 3501-mi. range associated with the original cruising speed (675.6 ft/s) in the performance example. For $V = 675.6$ ft/s, the specific range at average cruise gross weight is 21.70 mi./gal, while the total range divided by the total fuel consumed is 3501 mi. × 6.5 lb/gal ÷ 1050 lb fuel consumed = 21.67 mi./gal. Similarly, for $V = V_{\rm opt} = 596$ ft/s, $R_s = 22.05$ mi./gal, while $R/(W_i - W_f) = 22.03$ mi./gal. Thus it can be concluded that, in this example, maximization of R_s (at average cruise gross weight) is practically the same as maximizing cruising range (R).

Received Feb. 11, 1982.

*Benjamin H. Perkinson Professor of Engineering, School of Aerospace, Mechanical and Nuclear Engineering. Associate Fellow AIAA.